**UNIT-II**

**Solution of Linear and Non-linear Algebraic Equations**

**Gauss Elimination iterative Method:**

In this method the unknowns are eliminated successively and the system is reduced to an upper triangular system from which the unknowns are found by back substitution. The method is quite general and is well-adapted for computer operations. Here we shall explain it by considering a system of three equations for the sake of clarity.

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ( 1 )

Step 1: To eliminate x from second and third equations:

Assuming , we eliminate x from the second equation by subtracting times the first equation from the second equation. Similarly we eliminate x from third equation by subtracting times the first equation from the third equation. We thus ,get new system,

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ( 2 )

Here the first equation is called pivotal equation and is called the first pivot.

Step 2: To eliminate y from third equation:

Assuming , we eliminate y from the third equation by subtracting times the second equation from the third equation. We thus ,get new system,

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ( 3 )

Here the second equation is called pivotal equation and is the new pivot.

Step 3: To evaluate the unknowns:

The values of x , y , z are found from the reduced system ( 3 ) by back substitution.

Note: Clearly the method fails if any one of the pivots  ,  ,  become to zero. In such cases rewrite the equations in a different order so that the pivots are non – zero.

**Problems:**Solve the following system of equations by Gauss elimination method:

(1) x + 4y – z = -5 ; x + y – 6z = -12 ; 3x – y – z = 4 ;(A: x = 1-6479 , y = -1.1408 , z = 2.0845 )

(2) 10x – 7y + 3z + 5u = 6 ;-6x + 8y – z - 4u = 5 ; 3x + y + 4z + 11u = 2 ; 5x – 9y -2z + 4u = 7

(A: x = 5 , y = 4 , z = -7 , u =1 )

(3) 2x + y + z = 10 ; 3x + 2y +3z = 18 ; x + 4y + 9z =16

(4) 2x + 2y + z = 12 ; 3x + 2y +2z = 8 ; 5x +10y -8z =10

(5) 2x - y + 3z = 9 ; x + y +z = 6 ; x - y + z = 2

(6) 2x1 + 4x2 + x3 = 3 ; 3x1 + 2x2 -2x3 = -2 ; x1 – x2 + x3 = 6

(7) 5x1 + x2 + x3 +x4 = 4 ; x1 + 7x2 + x3 +x4 = 12 ; x1 + x2 +6 x3 +x4 = -5 ; x1 + x2 + x3 +4x4 = -6

**Gauss - Jordan iterative Method:**

This is a modification of the Gauss elimination method. In this method, elimination of unknowns is performed not in the equation below but in the equation above also, ultimately reducing the system to a diagonal matrix form. i.e., each equation involving only one unknown. From these equations the unknown x , y , z can be obtained readily.

**Problems:** Solve the following system of equations by Gauss Jordan method:

(1) x + y + z = 9 ; 2x - 3y + 4z = 13 ; 3x + 4y + 5z = 40

(2) 10x – 7y + 3z + 5u = 6 ; -6x + 8y – z - 4u = 5 ; 3x + y + 4z + 11u = 2 ; 5x – 9y -2z + 4u = 7

(3) 2x + 5y + 7z = 52 ;2x + y -z = 0 ; x + y + z = 9

(4) 2x - 3y + z = -1 ; x + 4y + 5z = 25 ; 3x - 4y + z = 2

(5) x + 3y + 3z = 16 ; x + 4y + 3z = 18 ; x + 3y + 4z = 19

(6) 2x + y + z = 10 ; 3x + 2y + 3z = 18 ; x + 4y + 9z = 16

**Factorization method ( or ) Crout’s method ( or ) L – U decomposition method:**

(1) 3x + 2y + 7z = 4 ; 2x + 3y + z = 5 ; 3x + 4y + z = 7

(2) 10x – 7y + 3z + 5u = 6 ; -6x + 8y – z - 4u = 5 ; 3x + y + 4z + 11u = 2 ; 5x – 9y -2z + 4u = 7

(3) 10x + y + z = 12 ; 2x + 10y + z = 13 ; 2x + 2y + 10z = 14

(4) x + 2y + 3z = 14 ; 2x + 3y + 4z = 20 ; 3x + 4y + z = 14

(5) 2x + 3y + z = 9 ; x + 2y + 3z = 6 ; 3x + y + 2z = 8

**Gauss – Seidel iteration method:**

An  matrix A is said to be “ diagonally dominated” if the absolute value of each leading diagonal element is greater than or equal to the sum of the absolute values of the remaining elements in that row.

In the system of simultaneous linear equations in n unknowns AX = B , if A is diagonally dominant then the system is said to be “ diagonal system “ .

Thus the system of equations,  is a diagonal system if 

Now we discuss Gauss – Seidel iteration method.

Consider the system of equations



Assume that the above system is diagonally system, then the above system rewritten as



We shall start the initial values for the variable  to be . Using the initial values in (1), (2), …. , (n) respectively we get .Puttig

In (1), (2), …. , (n) respectively we get the next approximations . In general the ( n+1 )th iteration are



Note: In solving a specific problem in the absence of any specific initial values for the variables we usually take the initial values of the variable to zero.

**Problems:** Solve the following system of equations by Gauss Seidel iteration method:

(1) 20x + y -2z = 17 ;3x + 20y - z = -18 ; 2x -3y + 20z = 25

(2) 10x + y - z = 11.19 ; x + 10y + z = 28.08 ; -x + y + 10z = 35.61

(3) 2x + y +6 z = 9 ;8x + 3y + 2z = 13 ; x + 5y + z = 7

(4) 28x + 4y - z = 32 ; x + 3y + 10z = 24 ; 2x + 17y + 4z = 35

(5) 10x + y + z = 12 ;2x + 10y + z = 24 ; 2x + 2y + 10z = 104

(6) 10x1 –2x2– x3 –x4 = 3-2x1+10x2 – x3 –x4 = 15; -x1 – x2+ 10x3 –2x4 = 27 ; -x1 – x2 – 2x3+10x4 = -9

**Newton – Raphson Method:**

Consider the equations  and 

Let  be an initial approximate solution of above system. Let  and  be the next approximate solution of above system.

Expand f and g by Taylor’s theorem for a function of two variables around the point , we have





( eliminating higher powers of h and k )

If  is a solution of given system then we have and then





Solve the above two equations for h and k. Now  and  will give a new approximation to the solution.

Note: We find the initial approximation by trial and error or by graphical method so that we can improve the accuracy and convergence is guaranteed.

**Problems:**

1. Solve the system of equationsand by Newton - Raphson iteration method with the approximation  ( x= 0.7719 , y = 0.4196 )
2. and with x = y = 2.828. (A: x = 3.162 , y= 2.45 )
3.  ,  with ( 3.5 , - 1.8 ) (A: x= 3.5844 , y = - 1.8481 )
4.  with ( 1 , 2 ) (A: x = 1.0828 , y = 1.9461 )